Introduction to Economics for Integrated Modeling

Michael Brady

Assistant Professor and Extension Economist
School of Economic Sciences
Washington State University

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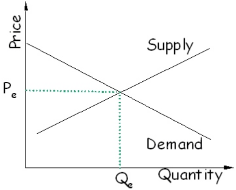
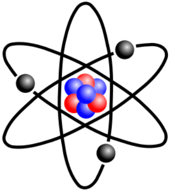
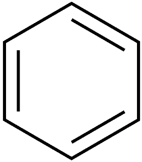
Models







The Supply and Demand Graph

- Supply and demand are mappings, curves, or relationships, not specific values.
- Quantity supplied and quantity demanded are specific amounts that correspond to how much of a good is supplied or consumed **at a given price**.
- **Equilibrium**: where supply and demand curves intersect. Rests on the market clearing assumption which states that price and quantity adjust so that markets clear.
- May seem like a simple and obvious idea but building models with endogenous prices is much trickier than taking prices as given.
- Also, a lot goes into proving the existence of an equilibrium. For example, von Neumann and Nash's use of Fixed Point Theorems laid the foundation for Game Theory.

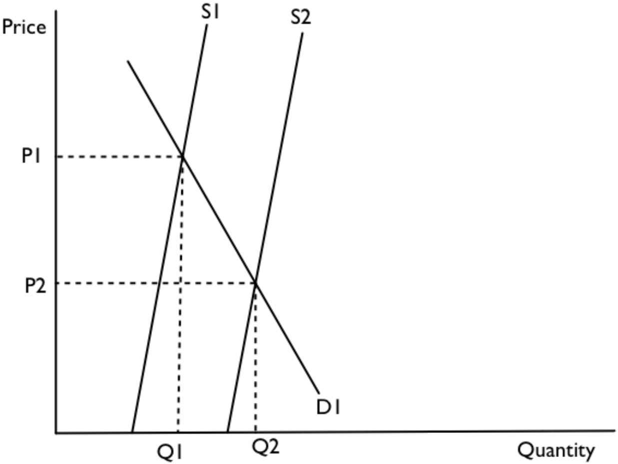
Elasticity

Elasticity

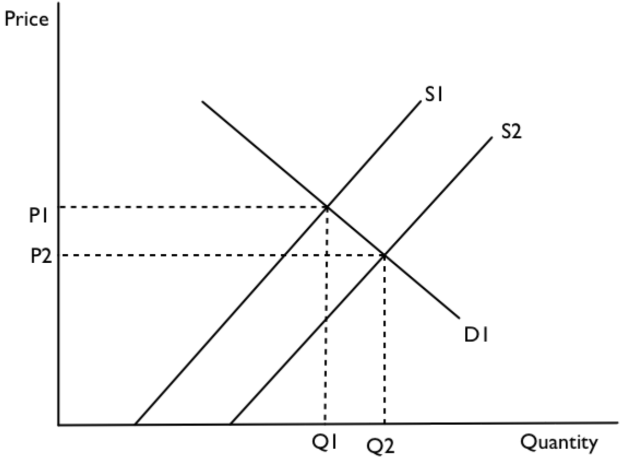
- Percent change in quantity supplied or demanded from a unit percent change in price.
- $\epsilon = \frac{p}{q}\frac{dq}{dp}$

The example given is a price elasticity, to be exact. Elasticities are also important for other economic variables like income.

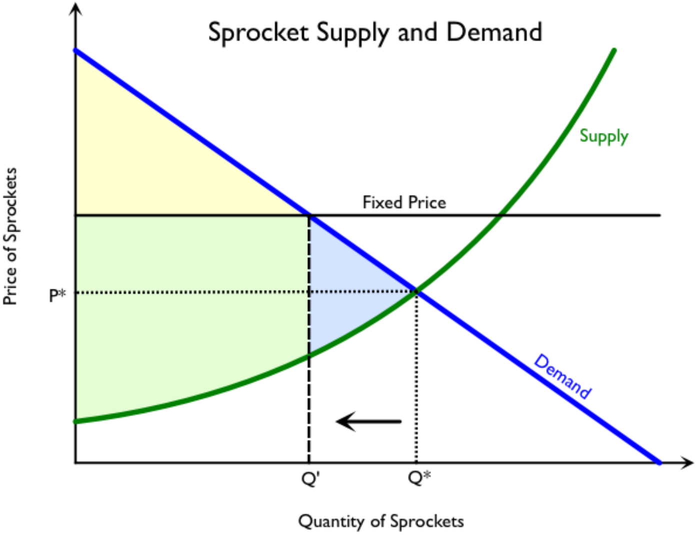
Inelastic supply and inelastic demand



Less inelastic supply and demand



Supply and demand with a price floor



Welfare

- Welfare analysis is the way that economists quantify and compare outcomes.
- Do not have a preference for the welfare of producers over consumers, or vice versa.
- Important to draw a distinction between political and economic goals.
- Economists can provide the menu of "efficient" outcomes but choosing between them is a political decision.

Modeling choices

- Single agent optimization
- Dynamics
- Risk and uncertainty
- Partial equilibrium
- General equilibrium

The Producer's Problem

Supply curve is derived from producers maximizing profit

$$\underset{\mathbf{x}\geq 0}{\text{maximize}} \quad py - \mathbf{rx}$$

- The case of one output quantity y with price p, and a vector of input quantities $\mathbf{x}$ and prices $\mathbf{r}$.
- Only constraint imposed at this point is nonnegativity for input quantities.
- Introduce the concept of the production function $y = f(\mathbf{x})$ and use π to denote profit.
- First order conditions (FOC)

$$\frac{\partial \pi}{\partial x_i} \Rightarrow p\frac{\partial f}{\partial x_i} = r_i$$

Optimal Production

Profit maximization achieved by choosing input quantity where the value of the marginal product is equal to marginal cost.

Marginal Value Product (or Value of the Marginal Product)

The value of the additional output generated by an additional unit of input (left hand side of FOC)

Marginal Cost

The cost of producing an additional unit of output (right hand side of FOC)

Second Order Conditions (SOC)

- Don't forget to check SOC to know whether you are at a max or min.
- As the old joke goes: Why did you pursue a PhD in economics? Because you forgot to check your second order conditions!
- Let $f_{ij} = \frac{\partial^2 f}{\partial x_i \partial x_j}$
- Check Hessian matrix for necessary (negative semi-definite) and sufficient (negative definite) conditions for a maximum.

$$H = \begin{pmatrix} pf_{11} & \cdots & pf_{1n} \\ \vdots & \ddots & \vdots \\ pf_{n1} & \cdots & pf_{nn} \end{pmatrix}$$

Supply

- Solve FOC for factor demands as a function of prices $x_i(p, r)$
- Factor is short for factors of production which is another way of saying inputs.
- Supply curve $\Rightarrow f(\mathbf{x}(p, \mathbf{r})) = y(p, \mathbf{r})$.
- Maximum profits at given prices $\pi(p, \mathbf{r}) = py(p, \mathbf{r})) - \mathbf{rx}(p, \mathbf{r})$.
- Have written the producer's problem as a function of exogenous variables.

Production Function

The production function

- Describes for each vector of inputs the amount of output that can be produced.
- $f : \Re_+^n \to \Re_+$ for the one output case.
- Continuous
- Strictly increasing
- Strictly quasiconcave on $\Re_+^n$
- $f(0) = 0$

Input Substitution

Isoquant $Q(y) \equiv \{\mathbf{x} \geq 0 | f(\mathbf{x}) = y\}$

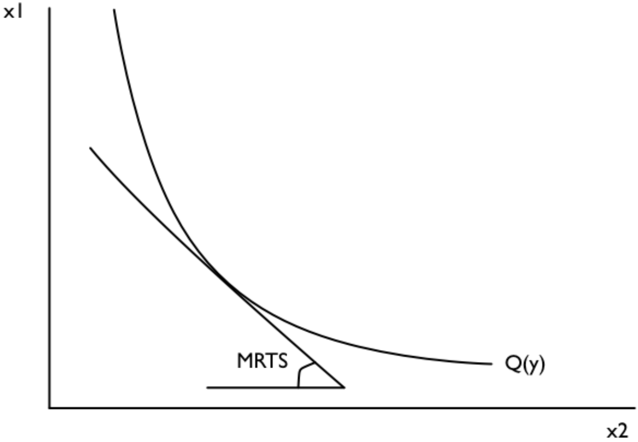
For any fixed level of output, y, the set of input vectors producing y.

Marginal rate of technical substitution (MRTS)

$$MRTS_{ij}(\mathbf{x}) = \frac{\frac{\partial f(\mathbf{x})}{\partial x_i}}{\frac{\partial f(\mathbf{x})}{\partial x_j}}$$

MRTS is the slope of the isoquant at some x_1 and x_2.

Input Substitution



Input Substitution

MRTS is useful but it is often better to have a unitless measure of substitutability.

Elasticity of substitution

$$\sigma_{ij} \equiv \frac{d\ln(x_j/x_i)}{d\ln(f_i(\mathbf{x})/f_j(\mathbf{x}))} = \frac{d(x_j/x_i)}{x_j/x_i}\frac{(f_i(\mathbf{x})/f_j(\mathbf{x}))}{d(f_i(\mathbf{x})/f_j(\mathbf{x}))}$$

- In words, percentage change in input proportion needed to maintain production from a 1% change in the MRTS.
- Assumption of quasiconcavity for the production function implies $\sigma_{ij} \geq 0$.
- $\sigma_{ij} = 0$ implies no substitution.

Scale and Scope of Production

Important to recognize the relationship between per unit cost and the size and scope of production.

Economies of scale

1. Constant returns to scale if $f(t\mathbf{x}) = tf(\mathbf{x})$ for all $t > 0$ and all $\mathbf{x}$.
2. Increasing returns to scale if $f(t\mathbf{x}) > tf(\mathbf{x})$ for all $t > 1$ and all $\mathbf{x}$.
3. Decreasing returns to scale if $f(t\mathbf{x}) < tf(\mathbf{x})$ for all $t > 1$ and all $\mathbf{x}$.

Economies of scope is same concept as scale but applies to the number of enterprises. For example, crop rotations reduce the per unit cost of production across all enterprises.

Example: CES production function

- CES stands for Constant Elasticity of Substitution
- $f(x) = (x_1^\rho + x_2^\rho)^{\beta/\rho}$
- $0 \neq \rho < 1$ and $\beta < 1$
- Take first order conditions
  1. $\partial\pi/\partial x_1 = p\beta(x_1^\rho + x_2^\rho)^{(\beta-\rho)/\rho} x_1^{\rho-1} - r_1 = 0$
  2. $\partial\pi/\partial x_2 = p\beta(x_1^\rho + x_2^\rho)^{(\beta-\rho)/\rho} x_2^{\rho-1} - r_2 = 0$
  3. CES production function is always increasing so assume an interior solution: $(x_1^\rho + x_2^\rho)^{\beta/\rho} - y = 0$.
- Take the ratio of 1 to 2 to get $x_1 = x_2(r_1/r2)^{1/(\rho-1)}$.
- Substituting into 3 and solving for y gives supply as a function of prices and parameters.
- Supply: $y = (\rho\beta)^{-\beta/(\beta-1)}(r_1^{\rho/(\rho-1)} + r_2^{\rho/(\rho-1)})^{\beta(\rho-1)/\rho(\beta-1)}$

Optimized production with a limited resource

- Many factors of production are limited in total available quantity.
- Operator labor time, water, land, financial resources.
- Cast the producer's problem as a constrained optimization problem.

General constrained optimization problem

$$\begin{aligned} \underset{\mathbf{x}\geq 0}{\text{maximize}} \quad & f(x_1, \ldots, x_n) \\ \text{subject to} \quad & g_1(\mathbf{x}) \leq b_1, \ldots, g_k(\mathbf{x}) \leq b_k \end{aligned}$$

Lagrangian for two inputs and one constraint

$$L(x_1, x_2, \lambda) \equiv f(x_1, x_2) - \lambda[g(x_1, x_2) - b]$$

Optimized production with a limited resource

- Analysis of the constrained optimization problem proceeds as is typical with this class of problems.
- The Lagrange multiplier, λ, has an important economic interpretation.
- If $\lambda > 0$ then the constraint is binding.
- The value of the multiplier represents how much the objective function changes if an additional unit of the input is made available.
- If the objective function is profit and the input is water then the multiplier represents the marginal value of water.
- Marginal value of water is how much the producer would be willing to pay for an additional unit of water if it were made available.

Allocating one output between two enterprise

- $f^i(x)$ denotes the production function for enterprise $i = 1, \ldots, I$
- Producer allocates finite input x across enterprises to equate marginal value products.
- Corresponds to $p^1 \frac{\partial f^1}{\partial x} = p^i \frac{\partial f^i}{\partial x} = \ldots = p^I \frac{\partial f^I}{\partial x}$

Optimal production with dynamics

- Up to this point have only considered the basic static model of supply.
- Most economic decisions are dynamic in that there are state variables that make current period choice sets a function of decisions made previous periods.
- Can break down irrigation throughout growing season on a weekly basis.
- Perennial crop production is inherently dynamic.
- Management of renewable and non-renewable natural resources is typically a dynamic optimization.

Optimal production with dynamics

- Can be cast as any combination of discrete/continuous time and discrete/continuous state.
- Three reasons why dynamic models of human decision making are difficult relative to dynamic models of physical or biological systems.
    - People are forward looking.
    - People are unpredictable.
    - People are heterogeneous in their cognition and preferences.

Basic types of dynamic models

- t=time; s_t=state of economic system; x_t=action; $f(s_t, x_t)$=reward.
- $V_t(s)$ denotes Bellman's Equation (1957) or value function.

Discrete time discrete space Markov decision model

$$V_t(s) = \max\{f(x,s) + \delta \sum P(s'|s,x)V_{t+1}(s')\}, s \in S, t = 1, 2, \dots, T$$

- Optimality is the development of a rule that optimally balances current against future value.
- Solved recursively but analysis also typically brings in dynamic-path and steady-state analysis.

Incorporating risk and uncertainty

- What sets agriculture apart from nearly most other sectors of the economy is the magnitude of stochastic shocks to production.
- This is due to factors like weather, pests, and the degree to which producers across the world are connected through global commodity markets.
- Often assumed that firms, in the general definition, are risk neutral.
- Most farm operations are sole proprietorships so they are not protected by limited liability.
- Risk aversion is incorporated into economic models by altering the producers objective function to include profit AND some other measure of variability of profits including standard deviation or max/min.
- For example, the objective function may be $\rho E(\pi) + (1 - \rho) V(\pi)$.
- Foundation for the enormous literature on decision making under risk is von Neumann Morganstern Expected Utility Theory.

Demand

- What determines the shape of the demand curve?
- Consumers purchase less of a good as the price increases.
- Continue to purchase as long as the benefit of the last unit is greater than the cost.
- The slope of the demand curve says a lot.
- Inelastic demand
  - Quantity demanded responds very little to price.
  - Demand curves are steeply sloped (near vertical) for goods that are necessities.
  - Gasoline, bread, rice.
- Elastic demand
  - Quantity demanded responds significantly to price.
  - Flat slope
  - Wine, steak, jewelry.

Markets

- To understand an entire market we need to bring together producers and consumers.
- Make assumptions about market structure to understand price determination.
- Market supply function $Q^s(p) \equiv \sum q^i(p, r)$ for $i = 1, \ldots, I$ producers.
- Model identifies market equilibrium price and quantity.
- Perfect Competition: assume that individual producers are price takers and choose optimal quantity to produce.
- Imperfect Competition: Producer with market power chooses price and quantity.

Welfare

- Welfare shifts analysis away from prediction and towards evaluation.
- Is it socially optimal to have only a small number of livestock processors?
- Was NAFTA bad for America?
- India recently put a ban on cotton exports which benefited textile mills but harmed farmers. Do the benefits outweigh the costs?
- Provides a foundation for policy such as anti-trust law.
- Total welfare is defined as the sum of producer and consumer surplus.

▸ Graphical example

General Equilibrium

- General equilibrium models try to explain an entire economy with multiple interacting markets.
- Provides a more complete picture by looking at how a chance in a particular market ripples throughout the entire economy including other industries and households.
- Tend to focus more on distributional questions than partial equilibrium models.

Model Integration Challenges

- Other models run over sub day time steps.
- Economic decisions are typically made over longer time horizons.
- Variable inputs can be adjusted monthly or maybe weekly.
- How should realizations of climate and water events be incorporated into expectations that economic agents have about the future?
- How does Rhessys clear over time?
- Grid cells can be modeled as individual optimizing agents but it is difficult to then relate to aggregate market effects.
- Can maximize production at the watershed level but that throws away a lot of biophysical model results.